RADIAL MEASURES OF PUBLIC SERVICES DEFICIT FOR REGIONAL ALLOCATION OF PUBLIC FUNDS

Version 29.12.99

Jaume Puig-Junoy *
Department of Economics and Business
Health and Economics Research Centre (CRES)
Universitat Pompeu Fabra
C/ Ramón Trias Fargas, 25-27, 34-08005 Barcelona (Spain)
Tel. 34.93.5421857 Fax 34.93.5421746
E-mail: jaume.puig @econ.upf.es.

(*)*Acknowledgements.- This study was funded by the Spanish Ministry of Health and Consumer Affairs, and by grant SEC 98-0296-C04-03 of the Spanish Comisión Interministerial para la Ciencia y la Tecnología. The views expressed are those of the author alone. The author thanks Professor C.A. Knox Lovell and Professor Guillem López-Casasnovas for his helpful comments and suggestions, and David Casado for his collaboration in the preparation and elaboration of the statistical data.*

RADIAL MEASURES OF PUBLIC SERVICES DEFICIT FOR REGIONAL ALLOCATION OF PUBLIC FUNDS

ABSTRACT

The goal of this paper is to present an optimal resource allocation model for the regional allocation of public service inputs. The proposed solution leads to maximise the relative public service availability in regions located below the best availability frontier, subject to exogenous budget restrictions and equality of access for equal need criteria (equity-based notion of regional *needs*). The construction of non-parametric deficit indicators is proposed for public service availability by a novel application of Data Envelopment Analysis (DEA) models, whose results offer advantages for the evaluation and improvement of decentralised public resource allocation systems. The method introduced in this paper has relevance as a resource allocation guide for the majority of services centrally funded by the public sector in a given country, such as health care, basic and higher education, citizen safety, justice, transportation, environmental protection, leisure, culture, housing and city planning, etc.

Keywords: Regional Allocation; Public Services; Equality of Access; Data Envelopment Analysis; Best Service Availability Frontier.

JEL Classification: H4, D6, I1.

1. INTRODUCTION

This paper analyses the problem of evaluating inequalities in the geographic distribution of public service inputs. The evaluation of regional inequalities in public service availability can have practical utility in the allocation of public financial resources in decentralised political systems. Issues of geographical equity arise from the way in which public resources are allocated. In many countries, public resources are not directed towards specific socio-economic groups, but are allocated geographically. From the point of view of resource allocation, the problem can be redefined as: "What are the most suitable criteria for adjusting regional capitative allocation, such that they will be coherent with equal opportunity of access to public services for equal need?" Systems with decentralised resource allocation have adopted different adjustment criteria to the *"needs"* that reflect distinct partial manifestations of need (variable selection and weights) which can be subject to discussion and manipulation, depending on the winning or losing position of each party involved. The result is usually an unstable equilibrium, which can reduce the credibility of the regional budget restrictions.

The goal of this paper is to present a resource allocation model for public regional services allocation whose solution leads to the maximisation of relative public service availability in the regions located below the best service availability, subject to an exogenous budget restriction and the equality of access for equal need criteria (equity-based notion of regional *needs*). Equality of access for equal need is the most commonly found definition of equity in public policy documents. The construction of non-parametric deficit indicators for public service availability is proposed, using a novel application of Data Envelopment Analysis (DEA) models, whose results offer advantages for evaluating and improving decentralised public resource allocation systems. This method has relevance as a resource allocation guide for the majority of services centrally funded by the public sector in a given country, such as health care, basic and higher education, citizen safety, justice, transportation, environmental protection, leisure, culture, housing and city planning, etc.

The structure of the paper is as follows: first (Section 2), the problem of measuring inequality of access is analysed in the framework of a welfare maximisation model. Second (Section 3), a linear programming method is proposed for evaluating the relative public service availability. Finally, Section 4 illustrates the effects of applying the proposed methods for the allocation of financial resources using the distribution of a health care levelling fund among the Spanish Autonomous Communities.

2. CONCEPTUAL FRAMEWORK

The optimal allocation of public funds would be that allocation which maximises the welfare associated with a given public service obtained with limited resources. To determine the optimal allocation conditions, the following steps must be followed. First, define the social evaluation function. Second, define the equivalent or adjusted service availability, or potential consumption. Third, define the reference vector to evaluate allocations to each region as the best service availability observed for the same need. And fourth, define the operative welfare measure and derive the conditions of the optimisation problem (resource allocation rule).

The model is based on the following assumptions. First, the social welfare associated with a public service depends exclusively on the availability of this service to the population located in an area (level of potential consumption in terms of possibility of access). Second, the average or representative consumers from each area are heterogeneous, but they only differ in their level of need (public service requirements). Third, the social welfare attached to a public service also depends on the equality of access to public services for the same need (equal opportunity to use the service, but not equality of use). Equality of access means that what is being provided is an equal availability, and equal opportunity to use the service. In setting up some way of allocating funds for public services geographically it seems desirable to tie access to the needs in different populations (Mooney, 1994). And fourth, at the moment of the access to the service in question, the consumers of each region have the right to the same level of service at the same monetary cost, which is usually negligible.

1. The social evaluation function (SEF).- The economy is composed of m distinct regions, with n being the number of different public service inputs that can potentially be accessed by individuals in each region. Individuals are characterised by their public service availability but they differ in their level of need. Each public service is used to produce a single type of service. The adjusted or equivalent quantity of service j available for the average or representative individual of the region i (per capita regional availability)[1] is denoted as x_{ij}, with $\mathbf{x}_i \in R^n_+$ being the adjusted or equivalent service availability of the i-th region. Potential consumption or service availability is adjusted because individuals are not homogeneous, differing in their level of need.

A social evaluation function (SEF) is a real valued function with the interpretation that provides the social or aggregate welfare from a normative point of view. A regional per capita SEF W(**x**) is adopted, which measures the social welfare associated with a public service allocation that gives rise to **x**:

$$W(x) = m m(x)[1 - \frac{1}{\ln m} T(x)], \quad (1)$$

where T(**x**) is the first Theil inequality index relative to the distribution of **x**. This function increases with the mean level of adjusted or equivalent service availability and decreases when the inequality of its distribution among the regions increases, which mirrors the Theil index. W(x) is characterised by the convenient simplification of making social welfare a function of only the mean of the distribution μ(x), and an inequality index. Dutta and Esteban (1992) established the formal conditions for an SEF to be expressed as a function only of the mean and an index of inequality. This SEF W(**x**) satisfies the axioms of differentiability, minimal equity, independence, homogeneity, and scale (Tomás and Villar, 1993). The SEF W(**x**) is

[1] Per capita measures of resources available in a region are used in the absence of individual level data.

defined following welfare economics literature which evaluates the social welfare of a population (see Ruiz-Castillo, 1995, for a review) taking into account a preference for efficiency (a preference for the greatest mean adjusted income, for example), and a preference for an egalitarian distribution which is expressed as a preference for the smallest possible value of a suitable index of inequality (as in Atkinson, 1970).

[Table 1]

2. The equivalent or adjusted service availability.- An alternative for homogenising potential consumption or service availability is to define an ideal per capita service availability vector $\mathbf{r}_i \in R^n_+$ for each region, given its level of heterogeneous needs. k_{ij} represents current non-adjusted per capita availability of service j in region i. Then, a per capita relative public service availability vector can be taken as a reference for defining adjusted or equivalent quantity of public service j available for the average individual of region i: $x_{ij} = k_{ij} / r_{ij}$.

The assumption that ideal per capita service availability is greater than or equal to current service availability for all m regions and all n services is adopted. That is to say, there is no legitimate reason to sustain that any region has public services in excess according to their needs. Then, x_{ij} ranges from 0 to 1. Given an ideal per capita availability vector of public services defined in relation to the needs, the adjusted or equivalent service availability is defined as the proportion of the current availability with respect to the ideal availability. When the current service availability coincides with the ideal, x_{ij} is equal to unity. When there is a relative deficit in the adjusted or equivalent quantity of public service j available for the average individual in region i, $x_{ij}<1$.

The regional per capita public service availability (k_{ij}) is the result of per capita financial resources devoted to service j in region i (M_{ij}), and also the result of the productive efficiency index of the service j in region i (θ_{ij}), where $\theta_{ij}=1$ if i is efficient and $\theta_{ij}>1$ if it is inefficient. Productive efficiency may be simply defined as the ratio between the observed functioning cost of per capita unit of service j in region i (γ_{ij}), and the minimum production cost of service j (c^*_j).

3. The best service availability frontier.- The ideal service availability vector $\mathbf{r}_i$ of each region is a function of the specific characteristics of the public service needs in that region (demographic, social and economic characteristics, etc.). Therefore, each region can be characterised by a semipositive vector of different availability of services: $\mathbf{k}_i = (k_{i1}, k_{i2},...,k_{in})$; and by a semipositive vector with characteristics specifically related to need: $\mathbf{n}_i = (n_{i1}, n_{i2},..., n_{is})$.

The ideal per capita service availability vector for region i is operatively defined as the greatest among those vectors for regions that have an equal or lesser need. In order to reach an operative measure of this definition, two consecutive steps are required. The first step is to define the *best service availability frontier for equal need*. And, the second is to identify the ideal service availability for region i with the corresponding availability of public services that would place it on the best service availability frontier.

The relationship between $\mathbf{k}_i$ and $\mathbf{n}_i$ for each region can be described using the graph GR = {(n,k): n is satisfied by k}. A family of available services that satisfy a level of need n can be defined as: $P(n) = \{k: (n,k) \in GR\}$, $n \in R_+^N$. The families of available services contain iso-need available services that can be defined as: Isoneed $P(n) = \{k: k \in P(n), \phi k \notin P(n), \phi \in (1,+\infty)\}$, $n \in R_+^N$; and these contain available services that reflect the best service availability frontier for equal need: Front $P(n) = \{k: k \in P(n), k' \notin P(n), k' \geq k\}$, $n \in R_+^N$.

A global measure of adjusted or equivalent ***public service availability*** (X_i^{RAD}) for the average individual in region i can be defined in a similar way to the radial measure of productive efficiency proposed by Debreu (1951) and Farrell (1957). This measure coincides with the equiproportional distance from the service availability and need vectors of region i to the best service availability frontier, that is: $X^{RAD}{}_i = 1/z_i^{RAD}$, where:

$$z^{RAD}(n,k) = \max\{\, f \mid fk \in P(n) \,\} \quad (2)$$

In expression (2), $z_i^{RAD}=1$ indicates that region i is located on the observed best service availability frontier for its level of need, while $z_i^{RAD}>1$ indicates the proportion that region i's vector of available public services should be increased in order to have the service availability equal to the region with more available services for the same level of need.

$z^{RAD}(n, k)$ is a radial measure, therefore $z(n, k)=1$ is a necessary but not sufficient condition for $(n, k) \in$ Front P(n). A non-proportional measure for the best service availability frontier for equal need would be equivalent to the following definition: the vector of available services is located on the best service availability frontier, given the vector of needs n, if and only if $n \in$ Front P(n). When there is a relative deficit in the adjusted or equivalent quantity of public service j available for the average individual in region i, $z_i^{RAD}>1$.

4. The optimal allocation of financial resources at a decentralised level.- The optimal regional resource allocation rule is obtained following three steps. First, defining an empirical welfare measure. Second, deriving an optimal resource allocation criterion. And third, deriving an optimal rule assuming that the results of step two do not produce an acceptable agreement for all regions.

(i) A social evaluation function $W(\mathbf{X}^{\mathbf{RAD}})$ is adopted, which measures the social welfare associated with a resource distribution that gives rise to $\mathbf{X}_i^{\mathbf{RAD}}$ (transformation of the original $\mathbf{k}$ allocation):

$$W((X_{RAD}) = mm(X_{RAD})[1 - \frac{1}{\ln m} T(X_{RAD})], \quad (3)$$

(ii) The problem of allocating a given volume of financial resources (M) in order to finance the decentralised provision of public services in the m regions consists of solving the following problem:

$$\text{Max } W(X_{RAD})$$
$$s.t. \sum_{i=1}^{m} M_i \leq M,$$
$$X_{RADi} \leq 1, \quad (4)$$

where M_i is the global financial allocation to region i. That is to say, the mechanism simply attempts to maximise the social evaluation function under the budget restriction and subject to the condition of not exceeding the ideal service availability.

The maximum of the function is obtained when it is found that, for the $X_i^{RAD\,*}$ value of each region, $X_i^{RAD\,*} = X_j^{RAD*}\ \forall$ i,j = 1,2,...,m. Which is equivalent to say that $X_i^{RAD\,*} = \mu^*$ for all i[2]. Then the following expression is obtained:

$$\sum_{i=1}^{m} M_i = M = \sum_{i=1}^{m} \sum_{j=1}^{n} (r_{ij} c_j^*)\, m^* \quad (5)$$

In consequence, under the hypothesis (i) $\theta_i = \theta_j\ \forall$ i,j = 1,2,...m ; and (ii) $\gamma_i=\gamma_j,\ \forall$i,j = 1,2,...,m[3]:

$$M_i^* = \sum_{j=1}^{n} r_{ij} c_j^* \frac{l}{m}$$

$$\text{where } l = \frac{M}{\sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij} c_j^*} \qquad (6)$$

The above expression indicates that the optimal financial allocation for the decentralised financing of public services is to share resources in such a way that all regions suffer the same proportional loss with respect to their ideal allocation. That is, it entails equalising X_i^{RAD}. This

[2] The monotony of W(X_i^{RAD}) and the transformation of k_{ij} in X_i^{RAD} imply that the restriction is saturated.

[3] The hypothesis of equal operating cost per unit of public service infrastructure endowment for all regions ($\gamma_i=\gamma_j$, " i,j = 1,2,...m) requires that this variable is adjusted for the regional differences in the input costs, if this is the case.

distribution rule means equal proportional loss with respect to the maximum observed service availability for equal need[4].

The proportional rule applied to the allocation of financial resources between regions presents two types of problems if the budget resources to be distributed are not sufficient to guarantee that all regions are located on the best service availability frontier (ideal consumer vector). In this case, the resource allocation problem can be characterised as a problem with unattainable objectives. All the need for public resources can not be met. The observed best service availability frontier for equal need is a relative concept that can shift to the extent to which the representative persons are more demanding.

Firstly, the application of the proportional rule can result in an optimal allocation M_i* that is lower than the current allocations (M_i<M_i*), and thus will be subject to debate. In this case, the optimal allocation does not produce an acceptable agreement for all the regions: in the solution of equal proportional loss, some regions are situated below the current level. A downward adjustment in the current service availability constitutes a threat to welfare that is difficult to accept.

Secondly, if the implausible hypothesis of identical productive efficiency for all regions is abandoned, c_j* being not directly observable by the financing agent or the social planner, it results in the distribution rule assigning a greater financial resources to regions with a lower productive efficiency. The allocation based on the equalisation of the relative public service availability penalises those regions that reduce the need in the most efficient way with the resources available. That is to say that the introduction of regional equality criteria in the allocation mechanism of a decentralised system modifies the incentives and is vulnerable to strategic manipulation. One effect is that the rule rewards inefficient production by assigning proportionally fewer resources to the more efficient regions. Another is that regions will have incentives to channel their public expenditures to objectives with less impact on the index of need, given that this will guarantee greater future funding.

A possible proposal for a *second best* solution that reduces the effect of the two problems mentioned above (those arising from the optimal solution to problem (4)) - and thus achieves agreement between the agents - is to add an equality of regional access for equal need domain restriction. This restriction consists of accepting a loss that is less than the proportional one obtained as the optimal solution to problem (4), when this loss would give rise to an unacceptable allocation for region i (for example, M_i* less than the current allocation), and proportionally equalise the deficit of those regions whose deficit is greater than the proportional solution for all the regions (Bossert, 1993; Herrero and Marco, 1993; Marco,

[4] The properties of this allocation rule are the following (Herrero and Villar, 1994): efficiency, symmetry, consistency, consistency in the case of reductions in non-financial objectives, independence of objectives achieved, homogeneity, uniformity in the case of budget restrictions, homogeneity for monotonic prices, weak homogeneity for monotonic prices and independence of proportional objectives.

1995).

This *second best* solution is equivalent to defining a resource allocation process in two stages. In the first stage, financial resources are assigned the value of M_i^1, where the M_i^1 allocation represents a reasonable agreement (consensus) between all the regions. Various possible alternatives, which differ in their contribution to the goal of equal access for equal need, can be imagined for the definition of the allocation in the first stage: for example, assignments based on (i) the *status quo*; (ii) some minimal objectives; (iii) the same quantity per capita; or (iv) the same adjusted quantity per capita, in which an agreement exists on the adjustment criteria. In the second stage a quantity M_i^2 is assigned such that:

$$\sum_{i=1}^{m} M_i^2 = M^2 = M - \sum_{i=1}^{m} M_i^1, \qquad (7)$$

and fulfilling the equal proportionality rule (6) defined with respect to the best service availability frontier. This *second best* solution allows the maximisation of welfare subject to the possibility of a feasible agreement between the regions. This compromise can act to the detriment of the grade of achievement of the objective of equality of access for equal need, but has the advantage of also reducing the productive inefficiency incentive as resources allocated in the first stage increases. The problem caused by the presence of different levels of efficiency at the regional level is restricted to the M_i^2 allocation. In this two-stage allocation the domain of the access equality objective is restricted, for example, to the differential rate of growth of the financial allocations, instead of affecting the global allocation level, as in the case of the solution of expression (6).

3. METHOD

In general, the use of representative indicators of the level of relative public service availability in decentralised financial resource allocation has led to the construction of synthetic indices for phenomena such as relative service availability or access difficulty. This approach can be characterised as an aggregation problem for which there are many partial manifestations observable, without there being consensus on: (a) the selection of the variables; (b) the weighting method; (c) the functional form of aggregation; and (d) the functional relationship between the synthetic indices and the necessary amount of financial resources.

The principal alternative methods to aggregation in a synthetic index, as found in the literature, can be classified into the three following groups: (a) *ad hoc* methods without any theoretical basis (i.e., Biehl, 1986), (b) methods based on the empirical relationship between utilisation and need (i.e., Carr-Hill et al, 1994; NHS, 1994), and (c) methods based on a descriptive statistical approach (i.e., Bosch and Escribano, 1988; Puig-Junoy and López, 1995). An approach that makes it possible to overcome the disadvantages of these methods consists of obtaining a measure of the relative public service availability in terms of expression (2). This expression presents the problem in a form which is can be solved with a new application of

DEA, a technique applied traditionally to measure the efficiency of decision-making units. We propose an adaptation of the computational techniques that have evolved in the DEA literature to the problem of finding a method for computing a public service deficit measure[5].

We wish to determine the per capita relative public service availability for each region, using the conceptual approach of the preceding section. In a context of multiple need variables and multiple resources (service availability), an index z_i^{RAD} is that of summed weighted needs, divided by summed weighted available services. Remember that the index z_i^{RAD} measures the proportion that region i's availability vector should be increased in order to have the service availability equal to that of the region with the best level of services for equal need. Consider a particular region, with subscript 0 ($\mathbf{n}_o$, $\mathbf{k}_o$). A mathematical programming formulation of the relative service availability problem asks what need and resource weights would make the z_i^{RAD} measure minimal. Thus, we may write:

$$\min \ \varnothing_o(n,u,u_*) = \frac{\sum_{l=1}^{s}(n_l n_{ol} + u_*)}{\sum_{j=1}^{n} u_j k_{oj}} \qquad (8)$$

$$s.t. \quad \frac{\sum_{l=1}^{s}(n_l n_{il} + u_*)}{\sum_{j=1}^{n} u_j k_{ij}} \geq 1, \quad i=1,2,..,o,...,m$$

$$n_l, u_j \geq 0$$

where **u** and **u** represent, respectively, the marginal social value of the public service needs associated with each specific characteristic, and the marginal social value of the public services. This problem looks for a combination of non-negative weights (multipliers) (υ_r, u_j) referring, respectively, to the observable variables of need and service availability in region o, which makes it possible to create the lowest possible index z_i^{RAD}; subject to a normalisation condition that indicates that no region, including 0, can have an index lower than one when using the same weights that are used in region 0. Additionally, the variable ν_*, which can take positive or negative values, expresses the possibility of variable returns (increasing, decreasing or constant) in the relationship between need and service availability.

This model seeks to minimise the ratio of weighted needs to weighted resources, for an arbitrary region, subject to the constraint that the same ratio for the other regions should not be lower than unity (which is the minimum value of the index z_i^{RAD}). By solving this problem m

[5] Banker (1993) showed that DEA was a maximum likelihood estimator and that the DEA estimators were consistent, establishing the asymptotic statistical properties of DEA. Korostelev, Simar and Tsybakov (1995) established that DEA was a maximum likelihood estimator of the boundary set, where the boundary is a convex and monotonic function of its arguments. They derive the rate of convergence of DEA estimators and show that no other estimator converges at a faster rate.

times, each time with a different region serving as the reference region 0, the best service availability hyperplane can be identified and measured, and each region's distance from it can be obtained. The need and resource weights chosen are those that minimise the distance between each region and the best service availability hyperplane. These weights have the economic interpretation of "shadow prices". One property of DEA that makes it particularly suitable for estimating relative public service availability is that it places no restriction on the functional forms of the best service availability frontier.

The non-linear ratio of expression (8) can be converted into a linear programming problem using the Charnes and Cooper transformation (1962), whose dual form is equivalent to the BCC DEA model, oriented toward *outputs* (Banker et al, 1984). And, then it allows to obtain a measure of the radial mean of relative deficit in the public service availability of a region:

$$z^{RAD}(n_o, k_o) = \max f \qquad (9)$$

$$\text{subject to} \qquad -n_{ol} + \sum_{i=1}^{m} n_{il}\, l_i \leq 0, \quad l = 1,\ldots,s$$

$$f k_{oj} - \sum_{i=1}^{m} k_{ij}\, l_i \leq 0, \quad j = 1,\ldots,n$$

$$\sum_{i=1}^{m} l_i = 1$$

$$l_i \geq 0$$

4. A CASE STUDY: THE SPANISH HEALTH CARE SYSTEM

This section presents the results of an empirical illustration of the methods proposed in this paper (linear problem 9), using the Spanish health care system at the level of each of its 17 Autonomous Communities (AC). Health care provision in Spain is a mix of public and private provision: 4/5 of total health care is publicly provided and 1/5 by the private sector. Spain spent 7.4% of its GDP on health in 1997. The Spanish General Health Care Law establishes the equality of access to public health care services and the correction of health care inequalities as objectives. Currently, the operating budget of public health services in the Spanish ACs tends to be based almost exclusively on population in communities that have decentralised their services, while they are based on historical expenditures in communities with centralised management. Nevertheless, it is reasonable to suppose that the need for health care infrastructure depends on a greater number of variables than simply the number of inhabitants. Furthermore, any financing system should take into account the current service availability resulting from the asymmetries in past investments in Spain.

Variable selection: data and sources. - A selection of measurable or observable variables that reflect the service availability or the need of each of the 17 ACs (m=17) contains information relative to the period 1985-1992. The information sources used were basically the following: (i) the survey of inpatient health care institutions that is published each year by the subgroup of variables (Everitt and Dunn, 1991), given that this variable is the most representative of the variability of the principal corresponding factor.

The use of information from the principal component analysis in the solution of problem (9) suggests the introduction of weighting restrictions (multipliers; υ_r, u_j) for the need and service availability indicators respectively, as a way to incorporate additional judgement into the DEA. The only restriction imposed on the weights in problem (9) is strict positivity, which makes it possible to evaluate each region in the best way. Absolute flexibility can lead to situating regions on the frontier by assigning unjustifiably high or low values to the weights of each variable.

In the case of the estimation of the best service availability frontier, there are two reasons that support the introduction of restrictions on the weights. First, the principal component analysis provides additional information about the relative importance of the different components (the factors are ordered according to the explicative capacity of the variance) that should be reflected in the construction of the relative service availability index. And, upon analysing the regional data, situations are often found in which the number of regions is significantly reduced in comparison to the number of representative variables of service availability and need, which leads to all the regions being placed on the frontier if the flexibility of the model is not limited.

The use of the first principal components (or observable variables most correlated with each component) introduces into this work the restriction that weights be decreasing once the variables are ranked according to the proportion of the variance explained. The following restrictions are imposed on the primal: $\alpha \leq u_i/u_{i+1} \leq \beta$ for i=1,...,t-1; and $\alpha \leq \upsilon_i/\upsilon_{i+1} \leq \beta$ for i=1,...,n-1, where α represents the lower limit of the ratio, which takes a unitary value, and β the upper limit.

Evaluation of the relative public service availability. - The empirical evaluation of the relative service availability measure (in linear problem 9) requires the identification of service and need variables used in the solution of the linear programming problems. According to the proposed procedure, the subgroup of need and service variables is selected in two steps: (i) the principal components are calculated until an eigenvalue of 0.7 is achieved, and (ii) the variable with the highest absolute coefficient in each component is identified.

Thus, 15 need variables (s=18) and 8 service or input variables (n=8) are used to solve the linear programming problems (Table 2). The need variables are the following (the principal component number is in parentheses): adjusted mortality rate for cerebrovascular diseases (1), adjusted rate for the population with permanent disability (2), adjusted mortality rate for malignant tumours (3), life expectancy with subjective poor health (4), infant mortality rate (5), percentage of households without toilet (6), rate of reported malaria cases per inhabitant (9), adjusted rate of male population with body mass index > 30 (10), adjusted mortality rate for malignant breast tumours (11), adjusted rate of population that have ever been smokers (12), reported cases of AIDS per 100,000 inhabitants (13), reported cases of whooping cough per

100,000 inhabitants (14) and adjusted mortality rate for pneumonia and infectious diseases in children under age 5 (15).

[Table 2]

For their part, the service availability variables are the following (the principal component number is in parentheses): operating rooms per inhabitant (1), auxiliary hospital personnel per hospital bed (2), beds in paediatric services per inhabitant (3), beds in long term care centres per inhabitant (4), beds in obstetric units per inhabitant (3), beds in gynaecological units per inhabitant (6), beds in burn units per inhabitant (7) and beds in neonatal intensive care units per inhabitant (8).

The subgroup of need and service variables that have been selected is the best summary of the variability of the entire variable set analysed with the least loss of information. Thus, the value of the selected variables resides in their capacity to reduce dimensionality, and not in the interpretative value of each one of them considered by itself or in isolation. The results of the estimation of the radial measures of service availability with restricted weights are shown in Table 3.

[Table 3]

Since our radial measures of public service availability do not explicitly include a noise term, the resulting deficit measures will incorporate any stochastic noise into the data. Consequently, the analysis below focuses on regional average scores, rather than index values for individual observations. To further reduce the influence of any noise that might be present, we employ the method presented by Wilson (1995) to detect outliers in the data. We have not found observations where the modified radial relative measure of service availability produces values of z_i^{RAD} >3 (cases where observations i are far beyond the frontier formed by the remaining observations).

The first column in Table 3 is the adjusted or equivalent aggregate measure of public health care services availability in each region. For example, Catalunya obtains an average score of 0.797 for the period 1985-1992. This implies that this region has an average level of heath care inputs or services which is equivalent to the 79,7% of the level of services available by those regions located in the best service availability frontier for equal need. Alternatively, Catalunya has to increase its level of health care services 1,255 times in order to reach the frontier. Or, in other words, this region presents a relative deficit in the availability of health care services when compared with the regions with the highest service availability among those with the highest service availability.

According to the computed scores for the relative radial measure, only 27 of the 136 observations (combinations of AC and year) analysed were on the best service availability frontier for equal need ($X_i^{RAD}=1$). The aggregation of the time series for each AC implies that the global set of observations be compared with the observation with the best service availability for equal need in the entire period of 1985-1992, and not just for a given year. The average score for the radial X_i^{RAD} index is 0.777, with a minimum of 0.505 and a maximum of 1[6].

The regions with the best relative service availability are Canarias, Castilla and León, Navarra and La Rioja. These are the regions with the largest number of observations located on the best service availability frontier. At the other end of the distribution are the regions with the lowest relative service availability for equal need (highest deficit): Baleares, Cantabria, Castilla-la Mancha and Comunidad Valenciana.

The estimation of the effects of considering the optimal allocation of financial resources, based on the equalisation of the relative radial score in 1992, was completed assuming that enough financial resources were allocated to the second stage to guarantee that the relative public service availability was, for example, no lower than 0.85 for all regions (X_i^{RAD} 0.85).

Then, we calculate the amount of financial resources required to satisfy this goal in the regions where $X_i^{RAD}<0.85$. This amount, M'_i , corresponds to the minimum production cost (c^*_j) of the level of services which allows a service availability with $X_i^{RAD}=0.85$. This level of service availability is obtained as the product of the current service level (k_{ij}) and the necessary increase from this level to the desired service availability ($z'^{RAD}_i=0.85/X_i^{RAD}$). c^*_j is not directly observable, and it is assumed to be identical in all regions (there are no variations in the regional prices of resources). It is also assumed that c^*_j is equal to the observed cost (γ_{ij}) corrected by an index of regional productive efficiency (θ_i). $\theta_i \geq 1$ is an index of productive efficiency for health services management in region i ($\theta_i=1$ if region i is efficient).

Then we calculate M'_i for those regions with $X_i^{RAD}<0.85$ as:

$$M'_i = \sum_{j=1}^{n} c^*_j \, z'^{RAD}_i \, k_{ij} \,, \qquad (10)$$

$$c^*_j = \gamma_{ij}\theta_i^{-1}$$

To establish the relationship between optimal allocation and the radial relative measures of service availability, the values of c^*_j (or of γ_{ij} and θ_i) must be available so that allocation is independent of the management efficiency level of each region. The absence of directly observable information on of c^*_j and θ_i gives rise to the proposal that they be estimated using

[6] Two non-radial measures have been calculated based on the arithmetic mean of the calculated relative service availability for each of the types of resources considered in this analysis. Similarities in index rankings across the two non-radial and the radial measures can be inferred from the matrix of Pearson product-moment correlation coefficients. All estimated coefficients are significantly different from zero at the 99% significance level.

the Least Absolute Value (LAV) method, a goal programming problem which is an alternative to the least square method, especially when the error distribution of a data set is different from the normal or has longer tails than the normal. With the variables representative of service availability used in this section, and being M_i the observed allocation of financial resources in region i, the following equation is defined:

$$M_i = \theta_i \sum_{j=1}^{8} c_j^* k_{ij} + \varepsilon_i , \quad i = 1,..,136 \quad (11)$$

where ε_i is the error component of the per capita expenditure in region i, and it is a two-sided distribution, with no assumption made about its form. To obtain the value of c^*_j (the minimum production cost per capita of service j in all regions) and θ_i (the productive efficiency index of health services in region i), which are not observable, it is proposed that the following problem be calculated using expenditure data adjusted for inflation using an index of health care prices for the years t=1990, 1991, and 1992[7]:

$$\text{Min} \sum_{i=1}^{17} \sum_{t=1990}^{1992} [d_{it}^+ + d_{it}^-] \quad (12)$$

$$s.t. \quad \sum_{j=1}^{8} c_j^* \theta_i k_{ij,t} + d_{it}^+ - d_{it}^- = M_{it}, i = 1,..,17$$

$$\theta_i \geq 1, i=1,..,17$$

$$d_{it}^+ \geq 0, d_{it}^- \geq 0, \forall i, \forall t$$

Given the regional allocation of financial resources in year t, what amount of additional resources, above the observed allocation, would be necessary to allocate to regions with a service availability level lower than 0.85 ($X_i^{RAD} < 0.85$) in order to allow them to reach this level? The results of problem (12), that is c^*_j and θ_i were used to simulate the desired distribution of resources to each of these regions (AC) in equation 10 (M'_i) taking into account regional population. The results are also presented in Table 3.

The third column in Table 3 presents the percentage of an additional global budget that should be distributed to each region in order to guarantee that no region has a relative public service availability lower than 0.85 in 1992. That is, that the distance from the region with the lowest relative availability to the best service availability frontier for the same need is no higher than 1/0.85.

In this simulation, regions that should receive an additional allocation from this equalisation fund (similar to a second stage in the allocation of resources) are those with $X_i^{RAD} < 0.85$ in 1992: Andalucía, Baleares, Cantabria, Castilla-la Mancha, C. Valenciana, Extremadura, Galicia and País Vasco. Using the information in the third column of Table 3, for example, Andalucía

[7] Information on regional financial allocations to AC's before 1990 is not available.

should receive the 36,8 percent of the amount of financial resources devoted to the regional equalisation fund, and Comunidad Valenciana the 30.4 percent.

According to the results presented in Table 3, it can be seen that the distribution of financial resources in the second stage, with a moderate re-equalisation goal, (i) requires a relatively small volume of additional financial resources, since it would be equivalent to no more than 10% of the total observed expenditure under the hypothesis of an equalisation goal at a relative public service availability level of 0.85, and (ii) the ACs that are located on the best service availability frontier or very close to it would not receive resources in this second stage.

5. DISCUSSION

This work attempts to evaluate the inequalities in the regional distribution of public service infrastructure, in relation to the concept of equality of access for equal need. In a model that maximises the social evaluation function associated with a public service it can be shown that the optimal allocation of resources requires that the resources be distributed in such a way that all regions suffer the same proportional loss with respect to the ideal allocation. The ideal allocation is defined in terms of the concept of the best service availability frontier for equal need. This frontier is formed by those observed combinations of need and service availability that comply with the condition that there is no greater service availability for each level of need. The equiproportional distance of each region from this frontier constitutes the relative public service availability.

We have applied the DEA method to determine the best service availability frontier for the same need. Our novel application of the DEA technique has been successful in illustrating a new method of dealing with inequality measurement in multidimensional observable variables of need and availability in public services.

The results of the theoretical model show how subjecting the global financial allocation of each region to the optimal condition can give rise to perverse incentives that degrade the efficient transformation of inputs into outputs. A second optimal solution for this manifestation of the conflict between equity and efficiency consists in a two stage allocation process, reserving the goal of equal access for equal need for the second stage, since it allocates resources based on the equality of relative public service availability.

The empirical evidence presented here has shown the potential for application to the case of the Spanish health care system, as well as the utility of proposed theoretically based methods to guide the decentralised allocation of the financial resources of a regional re-equalisation fund (second stage in the allocation process).

The static and deterministic character of the linear programming problem proposed according to the DEA assumption of no measurement error adds to the weakness of principal component analysis as an exclusively descriptive method. The measures should tend not to confuse measurement error with relative availability differences, transforming its character from deterministic to stochastic, and should tend to be defined for the panel data analysis. To further reduce noise, it would be useful to construct confidence intervals for the relative scores by bootstrapping the geometric means. However, some important problems remain to be solved in providing a theoretical basis for applying the bootstrapping methods to non-parametric estimation of frontier functions (Banker, 1996). Despite all this, the empirical simulation performed constitutes an example of the relevance of the method employed to deal with the questions presented in this work with greater theoretical and methodological rigor than in the previous literature on decentralised resource allocation criteria for the financing of public services.

REFERENCES

Atkinson, A. (1970), “On the Measurement of Inequality”, Journal of Economic Theory, 2, 244-263.

Banker, R.D. (1993), “Maximum Likelihood, Consistency and Data Envelopment Analysis: A Statistical Foundation”, Management Science 39, 1265-1273.

Banker, R.D. (1996), “Hypothesis Tests Using Data Envelopment Analysis”, Journal of Productivity Analysis, 7, 139-160.

Banker, R.D., Charnes, A. and Cooper, W.W. (1984), “Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis”, Management Science, 30, 9, 1078-1092.

Bielh, D. (ed.) (1986), “The contribution of infrastructure to the regional development”, Commission of the European Communities.

Bosch, A. and Escribano,C. (1988), “Regional Allocation of Public Funds: An Evaluation Index”, Environment and Planning, 20, 1323-1333.

Bossert, W. (1993), “An Alternative Solution for Bargaining Problems with Claims”, Mathematical Social Sciences, 25, 205-220.

Carr-Hill, R.A. et al (1994), “A Formula for Distributing NHS Revenues Based on Small Area Use of Hospital Beds”, Occasional Paper, Centre for Health Economics, University of York.

Charnes, A. and Cooper, W.W. (1962), “Programming with Linear Fractional Functionals”, Naval Research Logistics Quarterly, 9, 181-186.

Debreu, G. (1951), “The Coefficient of Resource Utilization”, Econometrica, 19 (3), 273-292.

Department of Health and Social Security (1976), “Sharing Resources for Health in England”, HMSO, London.

Dutta, B. and Esteban, J.M. (1992), “Social Welfare and Equality”, Social Choice and Welfare, 50, 49-68.

Everitt, B.S. and Dunn, G. (1991), “Applied Multivariate Data Analysis”, E. Arnold, London.

Farrell, M.J. (1957), “The Measurement of Productive Efficiency”, Journal of the Royal Statistical Society, Series A, General, 120, Part 3, 253-281.

Herrero, C. and Marco, M.C. (1993), “Rational Equal-loss Solutions for Bargaining Problems”, Mathematical Social Sciences, 26 (3), 273-286.

Herrero, C. and Villar, A. (1994), “La asistencia sanitaria como un problema de allocation en presencia de objetivos inalcanzables”, in López, G. (dir.), Análisis económico de la sanidad, Departamento de Sanidad y Seguridad Social, Generalitat de Catalunya, Barcelona.

Korostelev, A.P., Simar, L. and Tsybakov, A.B. (1995), “On Estimation of Monotone and Convex Boundaries”, Publications de l'Institut de Statistique de l'Université de Paris 39:1, 3-18.

Lovell, C.A.K. (1994), “Linear Programming Approaches to the Measurement and Analysis of Productive Efficiency”, Top, 2 (2), 175-248.

Marco, M.C. (1995), “Efficient Solutions for Bargaining Problems with Claims”, Mathematical Social Sciences, 30 (1), 57-69.

Mooney, G. (1994), “Key Issues in Health Economics”, Harvester Wheatsheaf, New York.

NHS (1994), “HCHS Revenue Resource Allocation. Weighted Capitation Formula”, October, London.

Puig-Junoy, J. and López, A. (1995), “Assessing health care infrastructure at the regional level: a statistical approach”, Applied Economic Letters, 2, 463-6.

Ruiz-Castillo, J. (1995), “Income Distribution and Social Welfare: A Review Essay”, Investigaciones Económicas, XIX (I), 3-34.

Tomás, J.M. and Villar, A. (1993), “La medición del bienestar mediante indicadores de ‘renta real’: Caracterización de un índice de bienestar tipo Theil”, Investigaciones Económicas, XVII (1), 165-173.

Wilson, P.W. (1995), “Detecting Influential Observations in Data Envelopment Analysis”, Journal of Productivity Analysis, 6, 27-45.

Table 1

Notation

Measure	Description
m	Number of regions (i=1,....,m)
n	Number of public service inputs or services (j=1,....,n)
s	Number of variables representing public service needs (l=1,...,s)
t	Year
k_{ij}	Current non-adjusted availability per person of service j in region i
r_{ij}	Ideal or desired per capita availability of public service j in region i, given its level of need
x_{ij}	Adjusted or equivalent quantity of public service j available for the average individual of the region i
n_{il} o	Variable or characteristic l representing public service needs per person in region i
M_{ij}	Per capita financial resources devoted to service j in region i
θ_{ij}	Productive efficiency index of the service j in region i
γ_{ij}	Observed functioning cost of per capita unit of service j in region i
c^{*}_{j}	Minimum production cost per capita of service j in all regions
T(X)	First Theil inequality index relative to the distribution of X
W(X)	Social welfare associated with a public service allocation that gives rise to X
X_i^{RAD}	Adjusted or equivalent global radial measure of public service availability in region i

Table 2

Descriptive Statistics for the Need and Service Variables

VARIABLES	MEAN	STDEV	SKEWNESS	KURTOSIS	MIN	MAX
NEED						
Adjusted death rate for cerebrovascular diseases	78.044	17.72	0.437	-0.576	47	125.4
Adjusted rate for population with permanent incapacity	140.44	22.19	-0.086	-1.410	105.2	173.6
Adjusted death rate for malignant tumours	151.94	11.01	-0.351	-0.092	120.9	172
Expected subjective poor health	22.71	2.57	0.375	-0.247	17.9	29.3
Infant mortality rate	8.32	1.84	0.988	1.412	4.9	15.9
Percentage of households without toilet	5.01	2.99	1.334	1.408	1.9	13.4
Rate of reported malaria cases per inhabitant	8.04	14.63	3.034	10.042	0	86
Adjusted rate of population that consumes alcohol	64.90	6.49	-0.541	0.745	44.5	77.8
Rate of reported tuberculosis cases per inhabitant	25.49	12.78	1.866	8.675	5.1	102.2
Adjusted rate of male population with body mass index > 30	8.44	2.59	0.693	-0.657	4.9	15
Adjusted mortality rate for malignant breast tumour	20.25	2.99	0.293	-0.309	14.3	28.9
Adjusted rate of population that have ever smoked	50.09	2.16	0.614	2.047	45.7	57.6
Reported cases of AIDS per 100,000 inhabitants	53.80	51.62	1.329	1.467	0	240
Reported cases of whooping cough per 100,000 inhabitants	58.57	72.88	2.585	8.020	0.2	439.2
Adjusted mortality rate for pneumonia and infectious diseases in children under 5 years	3.57	3.03	1.288	2.773	0	17.1
INPUTS						
Operating rooms, per inhabitant	60.80	9.36	0.785	2.140	42.34	100.87
Auxiliary personnel, per hospital bed	0.72	0.17	0.546	0.622	0.32	1.36
Beds in paediatric units, per million inhabitants	243.60	55.08	-0.149	-0.283	113.17	378.19
Beds in long term care units, per million inhabitants	127.42	157.47	2.131	4.656	0	777.22
Beds in obstetric units, per million inhabitants	174.23	37.30	2.209	11.679	110.92	414.82
Beds in gynaecological units, per million inhabitants	135.76	95.23	10.483	117.46	91.70	1199.39
Beds in burn units, per million inhabitants	3.31	4.92	2.239	5.915	0	24.12
Beds in neonatal intensive care units, per million inhabitants	4.64	6.15	2.503	7.167	0	30.89

Table 3
RELATIVE HEALTH CARE SERVICES AVAILABILITY AT THE REGIONAL LEVEL IN SPAIN

AUTONOMOUS COMMUNITY	*RADIAL SERVICE AVAILABILITY MEASURE* (X_i^{RAD}) (*)	*PROPORTION OF AN EQUALISATION FUND(**)*
ANDALUCIA	0.725	36.8
ARAGON	0.818	-
ASTURIAS	0.862	-
BALEARES	0.605	6.2
CANARIAS	0.993	-
CANTABRIA	0.632	1.6
CASTILLA-LA MANCHA	0.643	7.8
CASTILLA Y LEON	0.948	-
CATALUNYA	0.797	-
C. VALENCIANA	0.634	30.4
EXTREMADURA	0.772	0.6
GALICIA	0.690	11.7
MADRID	0.837	-
MURCIA	0.881	-
NAVARRA	0.976	-
PAIS VASCO	0.745	4.9
LA RIOJA	0.998	-

(*) Average for the period 1985-1992.

(**) Under the hypothesis of an equalisation goal at a relative public service availability level of 0.85 in 1992 ($X_i^{RAD} < 0.85$).

ANNEX

Table A.1
Health Care Service Variables

	VARIABLES
K1	Licensed doctors/population
K2	Licensed nurses/population
K3	Beds in Medical units/population
K4	Beds in surgical units/population
K5	Beds in obstetric units/population
K6	Beds in gynaecology units/population
K7	Beds in paediatric units/population
K8	Beds in psychiatric units/population
K9	Beds in tuberculosis units/population
K10	Beds in long term units/population
K11	Beds in intensive care units/population
K12	Beds in burn units/population
K13	Beds in neonatal intensive care units/population
K14	Operating rooms/population
K15	Physician/hospital bed
K16	Other health care personnel/hospital bed
K17	Other personnel/hospital bed
K18	Licensed pharmacists/population
K19	Licensed dentists and orthodontists/person

Table A.2
Variables representing Need of Health Care Services

	VARIABLES
N1	Unemployment rate
N2	Rate of days of limited primary activity per person, adjusted by age and sex
N3	Adjusted rate of bedridden days per person, adjusted by age and sex
N4	Adjusted rate of population with permanent disability
N5	Reported cases of tuberculosis per 100,000 inhabitants
N6	Reported cases of viral hepatitis per 100,000 inhabitants
N7	Reported cases of whooping cough per 100,000 inhabitants
N8	Reported cases of syphilis per 100,000 inhabitants
N9	Reported cases of gonorrhoea per 100,000 inhabitants
N10	Reported cases of AIDS per 100,000 inhabitants
N11	Rate of reported tetanus cases per inhabitant
N12	Rate of reported malaria cases per inhabitant
N13	Life expectancy at birth, both genders (inverted)
N14	Life expectancy at birth, men (inverted)
N15	Life expectancy at birth, women (inverted)
N16	Potential years of life lost per 1000 inhabitants, adjusted by age and sex
N17	Infant mortality rate
N18	Neonatal mortality rate
N19	Perinatal mortality rate
N20	Maternal mortality rate
N21	Adjusted mortality rate for cardiovascular diseases
N22	Adjusted mortality rate for ischaemic heart disease
N23	Adjusted mortality rate for cerebrovascular diseases
N24	Adjusted mortality rate for malignant tumours
N25	Adjusted mortality rate for malignant tumour of the trachea, bronchitis and lung
N26	Adjusted mortality rate for malignant tumour of the cervix
N27	Adjusted mortality rate for malignant breast tumour
N28	Adjusted mortality rate for external trauma and intoxication
N29	Adjusted mortality rate for traffic motor vehicle accidents

Table 2 (cont.)

N30	Frequency index of workplace accidents weighted by severity index
N31	Adjusted mortality rate for suicide and self inflicted wounds
N32	Illiteracy rate
N33	Rate of heavy smokers
N34	Adjusted rate of population that have ever smoked
N35	Percentage of births equal to or greater than 2500 grams
N36	Adjusted rate of sedentary population
N37	Adjusted rate of male population with a body mass index greater than 30
N38	Adjusted rate of female population with a body mass index greater than 30
N39	Adjusted rate of population that consumes alcohol
N40	Adjusted rate of heavy drinkers
N41	Number of admissions to ambulatory treatment for abuse or dependence on opiates or cocaine, per capita
N42	Percentage of primary households without running water
N43	Percentage of households without toilet
N44	Percentage of occupants of primary residences without telephone
N45	Percentage of population with ages between 0 and 4
N46	Percentage of population with ages between 65 and 74
N47	Percentage of population aged 75 or greater
N48	Subjective poor health expectancy
N49	Life expectancy in good health (inverted)
N50	Adjusted rate of mortality for pneumonia and infectious diseases in children under 5 years
N51	Incapacity free life expectancy (inverted))
N52	Percentage of population that evaluates their health as poor
N53	Percentage of population that evaluates their health as very poor